Scalar-Tensor Gravity and Quintessence

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Scalar fields with inverse power-law effective potentials may provide a negative pressure component to the energy density of the universe today, as required by cosmological observations. In order to be cosmologically relevant today, the scalar field should have a mass $m_\phi = O(10^{-33}\text{eV})$, thus potentially inducing sizable violations of the equivalence principle and space-time variations of the coupling constants. Scalar-tensor theories of gravity provide a framework for accommodating phenomenologically acceptable ultra-light scalar fields. We discuss non-minimally coupled scalar-tensor theories in which the scalar-matter coupling is a dynamical quantity. Two attractor mechanisms are operative at the same time: one towards the tracker solution, which accounts for the accelerated expansion of the Universe, and one towards general relativity, which makes the ultra-light scalar field phenomenologically safe today. As in usual tracker-field models, the late-time behavior is largely independent on the initial conditions. Strong distortions in the cosmic microwave background anisotropy spectra as well as in the matter power spectrum are expected.

The growing evidence for an accelerated expansion of the universe [1,2], together with the analyses on cluster mass distributions [3] and preliminary data on the position of the first Doppler peak in the cosmic microwave background anisotropies [4], strongly indicate that a large component of the energy density of the universe has negative pressure.

In a flat space-time, a cosmological constant Λ with $\Omega_\Lambda \simeq 0.7$ could well play the role of this unknown component. Alternatively, space-time dependent scalar fields have been considered, in what are usually called 'quintessence models'. In particular, models with inverse power-law effective potentials, $V(\Phi) = M^{4+m}\Phi^{-m}$, exhibit attractor solutions with negative pressure [5–9], so that the presently observed accelerated expansion of the universe might be ascribed to a scalar field which is still rolling down its potential. The good point about these models is the existence of the attractor, which makes the present-time behavior nearly independent on the initial conditions of the fields [7,8]. On the other hand, the field energy density scales with respect to that of the background according to the power-law

$$\frac{\rho_\Phi}{\rho_B} \sim a^{6\frac{1+w_B}{2+m}} \ , \qquad (1)$$

where the equation of state for the background is $w_B =$

0 (1/3) for matter (radiation). Then, assuming that the field reached the attractor solution by the time of equivalence between matter and radiation $(1+z_{eq} = 2.4 \cdot 10^4 \Omega_0 h_0^2)$ the above ratio has varied by a factor

$$z_{eq}^{6\frac{1+w_B}{2+m}}$$

since then. The explanation of why it is $\rho_\Phi \simeq \rho_B$ right today is the so called 'cosmic coincidence' problem. For moderate values of the exponent m the required fine-tuning on the mass parameter M in the scalar effective potential is of the same order of that needed in the case of a pure cosmological term ($m = 0$ in eq. (1)). Then, there is no clear improvement in this respect.

An even tougher problem emerges when the phenomenology of the scalar field Φ is taken into account. Indeed, the following relation holds on the attractor solution [7]

$$V'' = (9/2)(1 - \omega_\Phi^2)[(m + 1)/m]H^2 \ ,$$

which means that the scalar field is practically massless today, $m_\Phi \sim H_0 \simeq 10^{-33}\text{eV}$. If the most general couplings of Φ with the rest of the world are allowed, phenomenologically disastrous consequences are induced, like violations of the equivalence principle and time-dependence of gauge and gravitational constants on a time-scale $O(H_0^{-1})$ [10,11]. The former are strongly constrained by Eötvös type experiments to less than 10^{-12} level, whereas present results on the time variation of coupling constants give $|\dot{\alpha}/\alpha| < 6.7 \times 10^{-17} yr^{-1}$ for the electromagnetic coupling [12,13], $|\dot{G}_F/G_F| < 10^{-12} yr^{-1}$ for the Fermi constant [12] and $|\dot{G}/G| = (-0.2 \pm 1.0)10^{-11} yr^{-1}$ for the Newton constant [14].

Scalar-tensor theories of gravity (ST) represent a natural framework in which massless scalars may appear in the gravitational sector of the theory without being phenomenologically dangerous. In these theories the purely metric coupling of matter with gravity is preserved, thus ensuring the equivalence principle and the constancy of all non-gravitational coupling constants [10]. Moreover, as discussed in [15], a large class of these models exhibit an attractor mechanism towards general relativity (GR), that is, the expansion of the Universe during the matter dominated era tends to drive the scalar fields toward a state where the theory becomes indistinguishable from GR.

In this letter we will discuss quintessence in the framework of ST theories. We will identify a class of models in which two attractor mechanisms are operative at the same time: one towards the tracker solution, which accounts for the accelerated expansion of the Universe, and one towards GR, which makes the ultra-light scalar field phenomenologically safe. In these

models, the coupling between the scalar field and ordinary matter is a dynamical quantity which becomes smaller and smaller as the field rolls down its effective potential. This is the main difference with respect to previous works on ST theories as models for quintessence [16,17], where the attractor toward GR was not present and the coupling between the scalar field and matter had to be fixed to small values once for all in order to meet phenomenological constraints.

Moreover, we will find that during most of the matter-dominated era the ratio ρ_Φ/ρ_m will scale with the *logarithm* of a instead of the power-law of eq. (1). As a result, the variation of this ratio from equivalence to today is sizably reduced, thus alleviating the coincidence problem.

ST theories of gravity are defined by the action [10,15]

$$S = S_g + S_m \,, \qquad (2)$$

where

$$S_g = \frac{1}{16\pi} \int d^4x \sqrt{-\tilde{g}} \left[\Phi^2 \tilde{R} - 4\omega(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2\tilde{V}(\Phi) \right] \,, \qquad (3)$$

and the matter fields Ψ_m are coupled only to the metric tensor $\tilde{g}_{\mu\nu}$ and not to Φ, i.e. $S_m = S_m[\Psi_m, \tilde{g}_{\mu\nu}]$. $\tilde{R}$ is the Ricci scalar constructed from the physical metric $\tilde{g}_{\mu\nu}$. Each ST model is identified by the two functions $\omega(\Phi)$ and $\tilde{V}(\Phi)$. For instance, the well-known Jordan-Fierz-Brans-Dicke (JFBD) theory [18] corresponds to $\omega(\Phi) = \omega$ (constant) and $\tilde{V}(\Phi) = 0$. The matter energy-momentum tensor, $\tilde{T}^{\mu\nu} = 2/\sqrt{-\tilde{g}}\,\delta S_m/\delta\tilde{g}_{\mu\nu}$, is conserved, masses and non-gravitational couplings are time independent, and in a locally inertial frame non gravitational physics laws take their usual form. Thus, the 'Jordan' frame variables $\tilde{g}_{\mu\nu}$ and Φ are also denoted as the 'physical' ones in the literature. On the other hand, the equations of motion are rather cumbersome in this frame, as they mix spin-2 and spin-0 excitations. A more convenient formulation of the theory is obtained by defining two new gravitational field variables, $g_{\mu\nu}$ and ϕ by means of the conformal transformation

$$\begin{aligned} \tilde{g}_{\mu\nu} &\equiv A^2(\phi)g_{\mu\nu} \\ \Phi^2 &\equiv A^{-2}(\phi)G_*^{-1} \\ V(\phi) &\equiv A^4(\phi)\tilde{V}(\Phi) \\ \alpha(\phi) &\equiv \frac{d\log A(\phi)}{d\phi} \,. \end{aligned} \qquad (4)$$

Imposing the condition

$$\alpha^2(\phi) = \frac{1}{2\omega(\Phi) + 3} \,, \qquad (5)$$

the gravitational action in the 'Einstein frame' reads

$$S_g = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g}\,[R - 2g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2V(\phi)] \,, \qquad (6)$$

and the matter one now contains also the scalar field $S_m[\Psi_m, A^2(\phi)g_{\mu\nu}]$. In this frame masses and non-gravitational coupling constants are field-dependent, and the energy-momentum tensor of matter fields is not conserved separately, but only when summed with the scalar field one. On the other hand, the gravitational constant G_* is time-independent and the field equations have the simple form

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = -g_{\mu\nu}(g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi + V(\phi)) + 2\partial_\mu\phi\partial_\nu\phi + 8\pi G_* T_{\mu\nu}$$

$$\partial^2\phi - \tfrac{1}{2}\frac{\partial V}{\partial\phi} = -4\pi G_*\alpha(\phi)T \,, \qquad (7)$$

where $T_{\mu\nu}$ is the Einstein frame energy-momentum tensor, $T^{\mu\nu} = 2/\sqrt{-g}\,\delta S_m/\delta g_{\mu\nu}$.

The relevant point about the scalar field equation in (7) is that its source is given by the trace of the matter energy-momentum tensor, $T \equiv g^{\mu\nu}T_{\mu\nu}$, which implies the (weak) equivalence principle. Moreover, when $\alpha(\phi) = 0$ the scalar field is decoupled from ordinary matter and the ST theory is indistinguishable from ordinary GR. Indeed, at the post-newtonian level, the deviations from GR may be parameterized in terms of an effective field-dependent newtonian constant[1]

$$G = G(\phi) \equiv G_* A(\phi)^2 (1 + \alpha^2(\phi)) \,,$$

and two dimensionless parameters γ and β which, in the present theories turn out to be [10]

$$\gamma - 1 = -2\frac{\alpha^2}{1 + \alpha^2} \,, \quad \beta - 1 = \tfrac{1}{2}\frac{\kappa\alpha^2}{(1 + \alpha^2)^2} \,, \qquad (8)$$

where $\kappa = \partial\alpha/\partial\phi$.

The strongest bounds on the present values of the parameters α and κ come from solar system measurements and may be summarized as follows [15]

$$\alpha_0^2 < 10^{-3} \,, \quad [(1 + \kappa_0)\alpha_0^2] < 2.5 \times 10^{-3} \,. \qquad (9)$$

We next consider an homogeneous cosmological space-time

$$ds^2 = -dt^2 + a^2(t)\,dl^2 \,,$$

where the energy-momentum tensor admits the perfect-fluid representation

$$T^{\mu\nu} = (\rho + p)\,u^\mu u^\nu + p\,g^{\mu\nu} \,,$$

with $g_{\mu\nu}\,u^\mu u^\nu = -1$.

The Friedmann-Robertson-Walker (FRW) equations then take the form

$$-3\frac{\ddot{a}}{a} = 4\pi G_*\,(\rho + 3p) + 2\dot{\phi}^2 - V(\phi) \qquad (10)$$

$$3\left(\frac{\dot{a}}{a}\right)^2 + 3\frac{k}{a^2} = 8\pi G_*\rho + \dot{\phi}^2 + V(\phi) \qquad (11)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -4\pi G_*\,\alpha(\phi)(\rho - 3p) - \frac{1}{2}\frac{dV(\phi)}{d\phi} \,, \qquad (12)$$

with the Bianchi identity

$$d(\rho a^3) + p\,da^3 = (\rho - 3p)\,a^3 d\log A(\phi). \qquad (13)$$

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[1]Strictly speaking, this is only true for a massless field, but for any practical purpose it applies to our nearly massless scalars $(m_\phi \sim H_0^{-1})$ as well.

The physical proper time, scale factor, energy , and pressure, are related to their Einstein frame counterparts by the relations

$$d\widetilde{\tau} = A(\phi)d\tau, \quad \widetilde{a} = A(\phi)a, \quad \widetilde{\rho} = A(\phi)^{-4}\rho, \quad \widetilde{p} = A(\phi)^{-4}p.$$

Defining new variables

$$\chi \equiv \log\frac{a}{a_0}, \qquad \lambda \equiv \frac{V(\phi)}{8\pi G_* \rho}, \qquad w \equiv \frac{p}{\rho},$$

and setting $k = 0$ (flat space) the field equation of motion takes the more convenient form [15]

$$\frac{2}{3 - (\phi')^2}\ (1 + \lambda)\ \phi'' + [(1 - w) + 2\lambda]\ \phi' =$$
$$-\alpha(\phi)\ (1 - 3\ w) - \lambda\frac{d\log V(\phi)}{d\phi}, \qquad (14)$$

where primes denote derivation with respect to χ. Eq. (14) will be our master equation.

Each ST theory is specified by a particular choice for $\alpha(\phi)$ and $V(\phi)$. As already mentioned, a constant $\alpha(\phi) = \alpha$ and $V = 0$ select the traditional JFBD theory, in which no mechanism of attraction towards gravity is operative. In refs. [16,17] a non-zero potential with power-law behavior was added to this case, and the model was studied as a candidate for quintessence.

The mechanism of attraction towards GR can be illustrated by the simplest case $\alpha(\phi) = \beta\ \phi$, which was studied in refs. [15]. Choosing $V = 0$ eq. (14) takes the form of the equation of motion of a particle with velocity-dependent mass $m(\phi') = 2/(3 - (\phi')^2)$ in a parabolic potential $v = (1 - 3w)\ \beta\ \phi^2/2$, and subject to a damping force proportional to $(1 - w)$. Then, it is easy to realize that at late times the field ϕ will settle down at the minimum of the potential, $\phi = 0$, where $\alpha(\phi) = 0$ and the theory is indistinguishable from GR.

In order to have a model for quintessence, the late-time behavior of the field must be dominated by an effective potential with inverse-power law behavior. If this is the case, $\alpha(\phi)$ must decrease for large ϕ, unlike the behavior considered above. We then consider the class of ST theories defined by

$$\alpha(\phi) = -Be^{-\beta\phi}, \qquad V(\phi) = D\phi^{-m}, \qquad (15)$$

as models for quintessence.

From Bianchi identity we have the Einstein frame scaling laws

$$\rho_{\mathrm{rad}} \sim a^{-4} \qquad \rho_{\mathrm{mat}} \sim A(\phi)a^{-3},$$

so that the background equation of state turns out to be

$$w = \frac{1}{3}\frac{1}{1 + \frac{A(\phi)}{A(\phi_{eq})}e^{\chi - \chi_{eq}}}, \qquad (16)$$

$A(\phi_{eq})$ being the value of $A(\phi)$ at equivalence.

During radiation domination $w \simeq 1/3$, thus one might expect the equation of motion (14) to be insensitive to $\alpha(\phi)$. Actually, if one is interested in scalar fields with energy densities of the same order as matter today, then the scalar potential

term in the RHS of eq. (14) turns out to be subdominant with respect to $\alpha(\phi)(1 - 3w)$ during radiation domination and most of matter domination. In this regime, and neglecting ϕ'' with respect to ϕ', we find the approximate solution

$$\phi(a) \simeq \frac{1}{\beta}\log\left[\beta B\log\left(\frac{2}{3} + \frac{a}{a_{eq}}\right) + \mathrm{cost}\right], \qquad (17)$$

which is an attractor in field space. In deriving eq. (17) we have also assumed $A(\phi) \simeq A(\phi_{eq})$ which is always the case after nucleosynthesis, if the bound (22) below is satisfied.

Notice that this attractor is not the one that would have been obtained in the $\alpha(1 - 3w) \to 0$ limit, that is the well known tracker solution

$$\phi_{\mathrm{tr}} \sim a^{\frac{3(w_B + 1)}{m + 2}}, \qquad (18)$$

considered in [5–9], whose energy density scales according to eq. (1).

The energy density of the solution (17) is dominated by the kinetic term ($w_\phi \simeq 1$) and scales according to

$$\frac{\rho_\phi}{\rho_B} = \frac{1 + \lambda}{3 - (\phi')^2}(\phi')^2 + \lambda$$
$$\simeq \frac{1}{3}(\phi')^2 \sim \left(\frac{\frac{a}{a_{eq}}}{\frac{2}{3} + \frac{a}{a_{eq}}}\right)^2\left[\beta B\log\left(\frac{2}{3} + \frac{a}{a_{eq}}\right) + \mathrm{cost}\right]^{-2}. \quad (19)$$

Thus, the field evolution may be schematically divided in two regimes; at early times, during radiation domination and a large part of matter domination, it is $|\alpha(1 - 3w)| \gg |\lambda\,d\log V/d\phi|$, and the relevant attractor is approximated by eqs. (17), (19). Notice that after equivalence ($a > a_{eq}$) the field energy varies only logarithmically with respect to the background energy so, in this regime, the cosmic coincidence problem is parametrically alleviated.

At late times, the potential term starts to dominate and the solution is attracted towards the usual tracker with negative pressure, eqs. (18), (1), while the ST theory flows towards GR as $\alpha \to 0$.

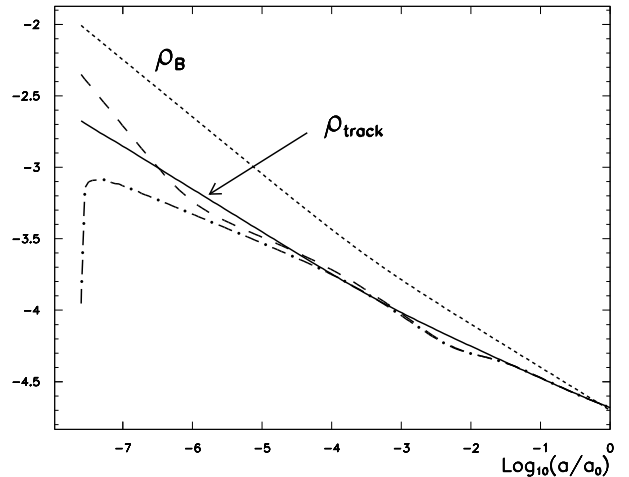FIG. 1. Energy densities (in $\log_{10}10$ scale) *vs.* Einstein frame scale factor for $B = 0.5$, $\beta = 4$ and $m = 6$. The short-dashed line is the background, whereas the long-dashed and dash-dotted are the field energy densities for two solutions with initial energy much larger and smaller than that of the tracker (18) (solid line), respectively.

In Fig. 1 the evolution of the energy density of typical solutions is shown as a function of $\log_{10}(a/a_0)$. The long-dashed and dash-dotted lines correspond to initial energies much larger and lower than the tracker solution (solid line), respectively. The latter has been obtained for $\alpha = 0$ and $\lambda \ll 1$ as in refs. [5–8]. The background energy density is plotted with the dashed line.

The potential term starts to dominate over the α-term only at late times $\log_{10}(a/a_0) \gtrsim -1.5$. Before that epoch the two solutions have already merged into the the attractor (17). As we see, the good point about the tracker solutions of refs. [5–8], namely the independence on the initial conditions, is preserved in this ST model. Here, differently from [5–9] it is achieved by means of the attractor (17) and not of the tracker (18). Eq. (18) becomes the relevant attractor only for $\log_{10}(a/a_0) \gtrsim -1.5$, when the potential term starts to dominate over the α one and the two solutions join the solid line.
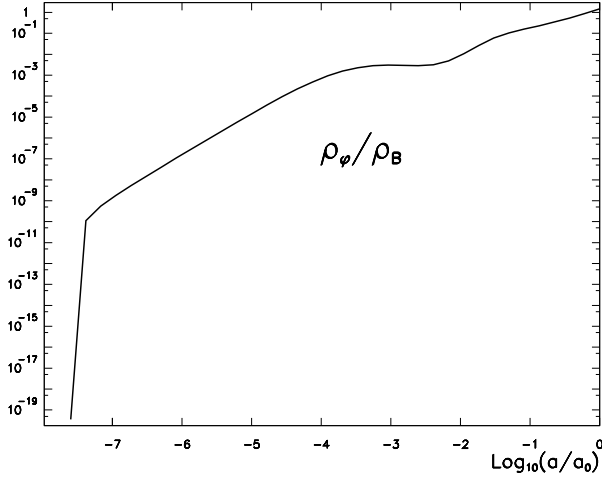

FIG. 2. Field to background energy ratio for the dash-dotted solution in Fig.1.

In Fig.2 we plot the field to background energy ratio for the dash-dotted solution of Fig.1. Notice the logarithmic behavior from equivalence ($\log_{10}(a/a_0) \simeq -4$) to the epoch in which the potential term starts to dominate.

The total equation of state in the physical frame, $\widetilde{W}_{tot} \equiv \widetilde{p}_{tot}/\widetilde{\rho}_{tot}$, is plotted in Fig. 3 versus the physical redshift $1 + z \equiv \tilde{a}/\tilde{a}_0$.
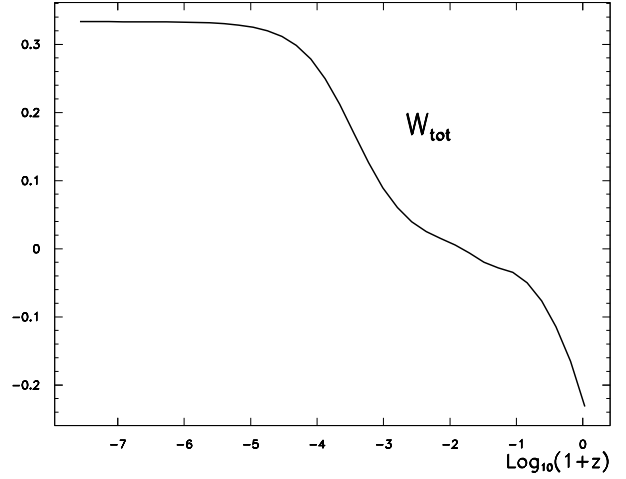

FIG. 3. The total equation of state (matter+scalar field) in the physical frame, for the dash-dotted solution in Fig.1.

Now we come to the phenomenological constraints on this ultra-light scalar. As we read from Fig. 4, the bounds coming from solar-system experiments, eq. (8) are largely satisfied by the present values of α^2, so that we don't expect any deviation from GR to be measured at present or in forthcoming experiments.

The strongest bounds come instead from nucleosynthesis. The variation of the (Jordan frame) Hubble parameter at nucleosynthesis induced by the time-dependent Newton constant is given by

$$\frac{\Delta \tilde{H}^2}{\tilde{H}^2} = 1 - \frac{\Phi_{nuc}^2}{\Phi_0^2} = 1 - \frac{A_0^2}{A_{nuc}^2} , \qquad (20)$$

and may be expressed in terms of the number of extra relativistic neutrino species as

$$\frac{\Delta \tilde{H}^2}{\tilde{H}^2} = \frac{7\Delta N/4}{10.75 + 7\Delta N/4} . \qquad (21)$$

Taking the 95% CL limit for ΔN, $\Delta N \leq 1$ [19] we get

$$\frac{A_0^2}{A_{nuc}^2} \geq 0.86 \qquad (95\% \text{ CL})$$

or, equivalently,

$$\alpha_0 - \alpha_{nuc} \leq 0.08\,\beta \qquad (95\% \text{ CL}), \qquad (22)$$

which gives constraints on the ratio B/β or on the value of the field at nucleosynthesis.

In Fig. 4 we plot α^2 for the dash-dotted solution of Fig. 1. As we anticipated, imposing the nucleosynthesis constrain (22) the post-newtonian bounds (8) turn out to be phenomenologically irrelevant. On the other hand, strong signatures of the present scenario are generally expected on the anisotropy spectra of the Cosmic Microwave Background (CMB) as well as in the matter power-spectrum. These issues have been studied in ref. [16] in the context of ST models which do not exhibit the attractor behavior towards GR considered here. The Induced Gravity (IG) model considered there is recovered in our language with a field-independent α, related to the parameter ξ of ref. [16] by

$$\alpha_{IG}^2 = \frac{2\xi}{1 + 6\xi} .$$

Being field independent, α_{IG}^2 is bounded by (9) at any epoch, and the resulting spectra distortions are at the some percent level in IG [16]. On the other hand in the present case we may have $\alpha^2 = O(10^{-2})$ at decoupling, as we see in Fig. 4. Looking at Fig. 6 of ref. [16] we can infer that such a large value may decrease the height of the first Doppler peak in the temperature anisotropy spectrum by a factor $\simeq 2$ and shift it to higher multipoles by $\Delta l \simeq 50$. Similar strong effects - well into the reach of future MAP and PLANCK CMB experiments - are predicted for the CMB polarization anisotropy and the matter power spectra.
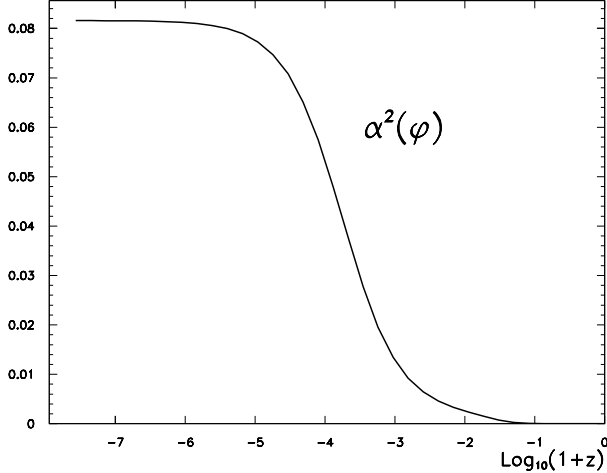

FIG. 4. The dynamical evolution of the matter-scalar coupling α in the physical frame, for the dash-dotted solution in Fig.1.

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